NOTATION

u, v, w, axial, radial, and tangential velocities; ε, scalar turbulent viscosity; L, mixing path; U_0, W_0, characteristic axial and tangential velocities at the outlet of the swirling units; η, Ω, degree and angle of initial swirling; Ψ, degree of blocking of channel; r, characteristic radius of curvature of annular channel; l, length of recirculation zone; $\xi = (y-R)/H$.

LITERATURE CITED

1. J. O. Hinze, Turbulence, McGraw-Hill, New York (1975).
2. G. N. Abramovich (ed.), Turbulent Jets of Air, Plasma, and Real Gas, Plenum Publ. (1969).
3. A. L. Dorfman, "Numerical solution of three-dimensional problem of the interaction of a system of jets in a carrier flow of viscous incompressible fluid," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 181-185 (1978).
4. A. L. Dorfman, "Explicit division scheme with application to the problem of three-dimensional interaction of jets of viscous incompressible fluid in a channel," in: Numerical Methods of Continuum Mechanics [in Russian], Vol. 9, VTs Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1978), No. 4, pp. 57-67.
5. A. G. Daikovskii and L. A. Chudov, "Effect of difference-scheme factors in calculating the wake behind a poorly streamlined body," in: Numerical Methods of Continuum Mechanics [in Russian], Vol. 6, VTs Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1975), No. 5, pp. 34-44.

MOTION AND MASS TRANSFER OF BUBBLES IN FLUIDIZED BED

V. A. Borodulya, Yu. A. Buevich, and V. I. Dikalenko

UDC 66.096.5

Data on the rise velocity and mass-transfer coefficient of single gas bubbles in beds of particles of various dimensions in a state close to minimum fluidization are obtained and discussed.

Mixing processes in chemical reactors and other equipment with an inhomogeneous fluidized bed depend on the behavior of gas bubbles and their interaction with the dense phase of the bed, and to a considerable extent determine the efficiency of their operation.

Despite the considerable amount of factual material on the properties of bubbles and their effect on the distribution of the gas flows and particles obtained in recent years both in laboratories and in industrial conditions (see [1], for example), the complete picture of bubble behavior in the bed remains unclear in many important details. This is because many heterogeneous physical factors affect the behavior of the bubbles in a two-phase medium and because most experiments refer to a narrow range of variation of the determining parameters. Therefore, the present work is a systematic investigation of the dependence of the two most important bubble characteristics — the rise velocity and the mass-transfer coefficient with the dense phase of the bed — on only two parameters: the dimensions of the bubble itself and of the particles of which it is composed.

Experimental Method

The basic features of the apparatus used were described in [2]. Various fractions of quartz sand (see Table 1) were fluidized by air at the same temperature in a column of cross section 0.2 × 0.2 m; the height of the motionless filling in all experiments was 0.5 m. Tracing gas (CO_2) was introduced into a bed in a state close to minimum fluidization through a pipe of diameter 0.004 m running along the central axis of the column at a height of 0.04 m above the gas-distribution plane and through a system of reducers and an electromagnetic valve. The volume of the bubble forming was regulated by changing the duration of the voltage pulse supplied to the electromagnetic pulse from a monovibrator and the excess gas pres-

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Ural State University, Sverdlovsk.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 4, pp. 678-689, October, 1981. Original article submitted September 30, 1980.

0022-0841/81/4104-1123$07.50

TABLE 1. Characteristics of Quartz—Sand Fractions Used in Experiments

Fraction composition $2a \cdot 10^3$, m	Content, wt.%	Mean diameter $2\langle a\rangle \cdot 10^3$, m	Density $\rho_p \cdot 10^{-3}$, kg/m^3	Fluidization rate $u_* \cdot 10^2$, m/sec
1,0—2,5 0,63—1,0 0,4—0,63 0,315—0,4 0,2—0,315	1,30 46,9 50,7 0,008 0,003	0,63	2,6	24,3
0,4—0,63	100	0,51	2,6	21,2
0,4—0,63 0,315—0,4 0,2—0,315 0,16—0,2 0,1—0,16 0,063—0,1	1,4 3,4 63,2 11,2 10,5 0,9	0,25	2,6	5,6
0,2—0,315	100	0,25	2,6	5,8
0,16—0,2	100	0,18	2,6	3,1
0,1—0,16	100	0,13	2,6	1,8

sure at the valve. Gas samples were taken automatically from the bubbles rising along the column axis, in accordance with the signals from photosensors positioned 0.240 and 0.436 m from the gas distributor. Use was made of a modification of the method of automatic selection adopted earlier in [3] in studying the mass and heat transfer in a two-dimensional fluidized bed.

The sample-selection system consisted of a sensitive element (a photodiode with a lamp), a photopulse amplifier, and an actuating device associated with the plunger of a syringe. When the bubble reaches the sensor, which until then has been obscured by particles, the diode is illuminated, and the resulting signal is amplified and fed to the electromagnetic relay of the actuating device. The relay triggers, and the syringe plunger is set in motion by the elastic force of a rubber band connected to the plunger. Gas from the bubble is drawn into the syringe through a needle of internal diameter 0.001 m, and the CO_2 content in the sample was determined by means of an analysis on an LKhM 7A chromatograph.

At the same time as the gas samples were collected, the pulsed signals of the sensors formed by the passing bubbles were recorded on an N-327/3 automatic recorder. The vertical bubble dimension $2R^*$ and the velocity U of bubble motion were determined, as in [3, 4], from known quantities: the distance between the sensors Δl, the distance between pulses on the automatic-recorder tape δl, the velocity of tape motion v, and the mean pulse width δ:

$$U = v\Delta l/\delta l, \quad 2R^* = U\delta/v. \tag{1}$$

The frontal diameter 2R of the bubble was determined from its vertical dimension $2R^*$ by means of a calculation based on the usual assumption that the bubbles take the form of a sphere of radius R with an ejected rear segment whose volume is 25% of the total sphere volume [1, 4].

In view of the inertia of the device for sample selection, it was necessary to find the upward displacement of the vertical coordinates of the sampling point with respect to the coordinates of the sensors, so that gas from the central part of the bubble entered the syringe. To this end, experiments were formulated on an apparatus with a two-dimensional bed, in which a cinerecording of the ejected bubbles was made. Hence, the dependence of the displacement on the bubble rise velocity in the two-dimensional bed was determined, and from this the necessary lag time of sample selection was found; then, taking account of the bubble velocity in the three-dimensional bed, the required value of the given displacement with respect to the sensors was established. In the experiments here described it was 0.03-0.05 m.

The mass-transfer coefficient k referred to unit surface area of bubble appears in the mass-transfer balance equation

$$V\frac{dc}{dt} = -kSc \tag{2}$$

TABLE 2. Literature Data on the Rise Velocity of Single Bubbles in a Fluidized Bed

Source	Particles	Particle diameter, mm	Column size, cm	Measurement method	K	Correlating equation, U
[9]	Glass balls Quartz sand Turnip seeds	0,15 0,4 1,7	∅7,5 ∅7,5 ∅14,6	Capacity sensor	0,61	$\sim \sqrt{g}\, V^{1/6}$
[10]	Quartz sand	0,15—0,2	42×42	Capacity sensor	0,61	$\sim \sqrt{g}\, V^{1/6}$
[11]	Glass balls Silver powder Acrylate Catalyst Magnesite Coal particles	0,06—0,55 0,07—0,50 0,052 0,121 0,24 0,41	∅14	X-ray cinerecording	0,84—1,0 0,87—1,2 0,92 0,88 0,94 0,96	$\sim \sqrt{gR}$
[12]	Glass balls PVC	0,125—0,15 0,125—0,15	10×10	Capacity sensor, cinerecording	0,88	$\sim \sqrt{gR}$
[13]	Silica gel	0,15—0,21	∅10	Photosensor	0,79	$\sim \sqrt{2gR^*}$
[14]	Coke particles	0,344 0,154 0,086	∅10	Electrical-conductivity sensor	0,956	$\sqrt{g}\, V^{1/6}$
[4]	Glass balls Quartz sand	0,3—0,4 0,063—0,015	14,4×29,5	X-ray cinerecording	0,79—0,86	$\sqrt{gR}$
[15]	Alumosilicate catalyst	0,17—0,35	∅35	Special cinerecording	0,77	$\sim \sqrt{gR}$
[16]	Alumina	1,0—2,0	61×61	Inductive probe, cinerecording of the surface	0,67	$\sim V^{1/6}$

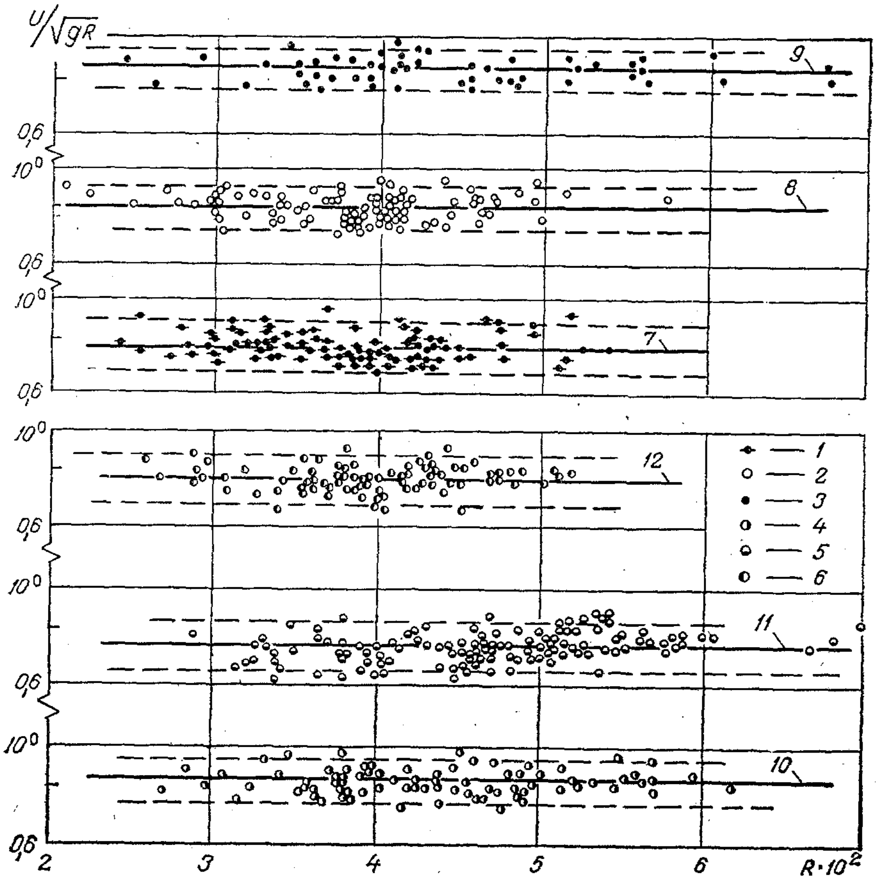

Fig. 1. Dependence of velocity coefficient K on bubble radius R in beds of particles of different size; the continuous curves correspond to mean values of K and the dashed curves mark the boundaries of the 95% confidence interval: 1-3 and 4-6) experimental values of $K = U/\sqrt{gR}$, respectively, for $2a = (0.1\text{–}0.16)\cdot 10^{-3}$, $(0.16\text{–}0.2)\cdot 10^{-3}$, and $(0.2\text{–}0.315)\cdot 10^{-3}$ m; $2\langle a\rangle = 0.25\cdot 10^{-3}$ m; $2a = (0.4\text{–}0.63)\cdot 10^{-3}$ and $2\langle a\rangle = 0.63\cdot 10^{-3}$ m; 7-9 and 10-12) mean values of K = 0.77; 0.81; 0.85; 0.82; 0.74; 0.77 for the corresponding particle sizes. $R\cdot 10^2$, m.

(the carbon dioxide concentration in the dense phase was assumed to be zero). Setting f = 0.25, and using the equation of bubble motion $dl = U dt$, it follows that

$$k = \frac{2R^*U}{5.35\Delta l} \ln \frac{c_1}{c_2}. \tag{3}$$

If there is a stable cloud of closed gas circulation around the bubble, the main mass-transfer resistance is concentrated at the external boundary of the cloud, and the most physically meaningful quantity is the mass-transfer coefficient k' referred to unit surface area of a sphere of radius R'. In this case

$$\left[(1-f)\frac{4}{3}\pi R^3 + \varepsilon\left\{\frac{4}{3}\pi R'^3 - (1-f)\frac{4}{3}\pi R^3\right\}\right]\frac{dc}{dt} = -k'4\pi R'^2 c, \tag{4}$$

and hence after integration

$$k' = k\left[(1-f)(1-\varepsilon) + \varepsilon\left(1+\frac{2u}{U}\right)\left(1-\frac{u}{U}\right)^{-1}\right]. \tag{5}$$

The use of beds close to minimum fluidization allowed the influence of the gas flow directed into the growing bubble on the rise velocity and the mass transfer to the eliminated; the importance of this influence was noted in [5, 6]. Within the limits of experimental error, the bubbles retained the same volume over the period in which they crossed the distance between the sensors.

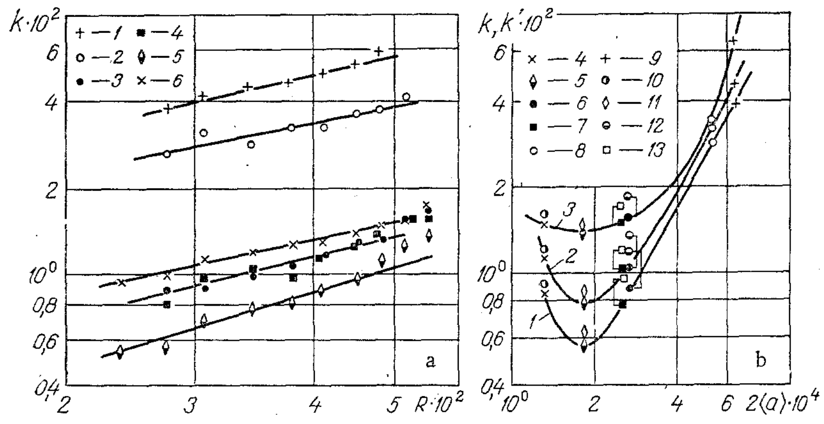

Fig. 2. Dependence of the mass-transfer coefficients k, k': a) on the bubble radius R in beds of particles of different size for $2\langle a\rangle = 0.63\cdot10^{-3}$ (1) and $0.25\cdot10^{-3}$ m (4) and for $2a = (0.4-0.63)\cdot10^{-3}$ (2), $(0.2-0.315)\cdot10^{-3}$ (3), $(0.16-0.2)\cdot10^{-3}$ (5), and $(0.1-0.16)\cdot10^{-3}$ m (6); the continuous curves correspond to the mean values of k for the corresponding particle sizes; b) on the diameter of the bed particles $2\langle a\rangle$ for different bubble sizes: $2R^*=$ 3.75 (1), 5.25 (2), and 7.25 cm (3). The points show experimental values of k for $2a = (0.1-0.16)\cdot10^{-3}$ (4), $(0.16-0.2)\cdot10^{-3}$ (5), $(0.2-0.315)\cdot10^{-3}$ (6) and $(0.4-0.63)\cdot10^{-3}$ m (8) and for $2\langle a\rangle = 0.25\cdot10^{-3}$ (7) and $0.63\cdot10^{-3}$ m (9). k, $k'\cdot10^2$, m/sec.

It should be emphasized that considerable scatter was observed in the results of identical experiments; this was due not so much to the usual random errors of the measurements as to the presence of expressed pulsations of the form of the rising bubbles accompanying the pulsations of their velocity. Such pulsations have been observed more than once on earlier occasions [1]. In the experiments, a sufficient number of repetitions were accumulated to ensure reliability corresponding to the 95% confidence level.

Rise Velocity

The results on the rise velocity of the bubbles are presented in Fig. 1. They are correlated fairly well by the well-known formula

$$U = K\sqrt{gR}. \tag{6}$$

According to the Davis–Taylor formula, it would be expected that $K \approx 0.67$; in [7], on the basis of empirical data analyzed in [8], it was assumed that $K \approx 0.9\,(1-f)^{1/6}$, which is equal to 0.86 when f = 0.25. In fact, the "velocity coefficient" is markedly higher than 0.67, which corresponds qualitatively to the model in [5], but nevertheless it is not so high as would follow from the data collected in [1]. It is possible that the value $K \approx 0.85-1.20$ [1] arises in a number of cases because in fact it was not the velocity of single bubbles that was measured, but the higher rise velocity of a group of bubbles.

In addition, Table 2 gives data of various authors relating to single bubbles. These are characterized by a large scatter, which may be associated with unavoidable errors in the experimental determination of quantities such as $2R^*$ or V, and with the assumptions made in calculating the coefficient K on the basis of these data.

It may be concluded from the data in Fig. 1 that the mean value of the velocity coefficient at first rises with increase in particle size in the bed, which agrees with what is expected from [5], but then begins to fall. However, the scatter of the experimental points and the absence, in the main, of simultaneous measures of the bubble form characteristics prevent any completely firm conclusions from being drawn on the reasons for this dependence. Thus, according to [5], the rise velocity should depend both directly on u_* and also on f, which itself depends on the phase properties of the bed and, in particular, on u_* (hypotheses on the influence of the relative volume of the bubble tail filled by particles on U were introduced earlier in [1]). Elucidation of the question of the relation between the rise velocity and the phase characteristics of the bed is very important for the formulation of the physical model of a bubble in a two-phase medium. Therefore, the formulation of more

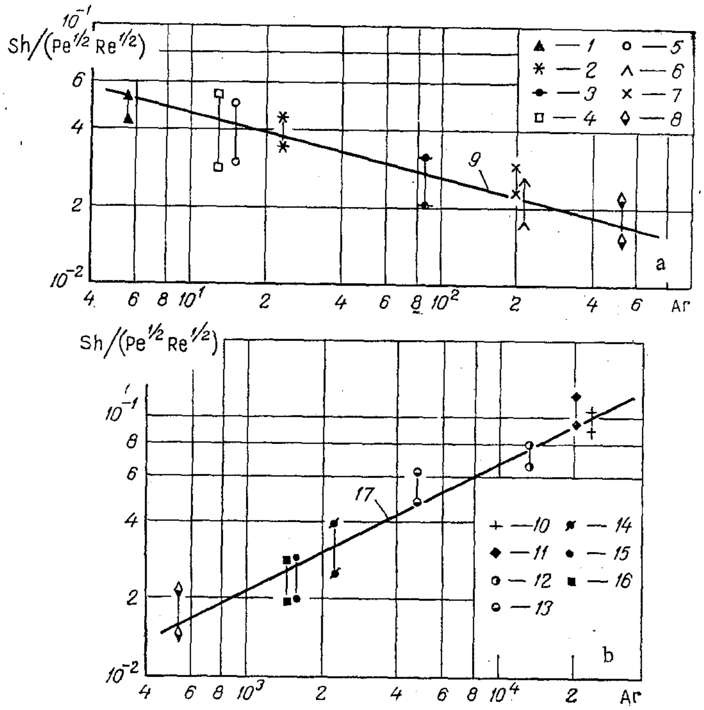

Fig. 3. Dependence of the parameter complex $Sh/\sqrt{PeRe}$ characterizing the rate of bubble mass transfer with the dense phase of the bed on Ar for particles diameter $2\langle a\rangle < 0.2 \cdot 10^{-3}$ m (a) and of diameter $0.15 \cdot 10^{-3} < 2 \cdot \langle a\rangle < 0.63 \cdot 10^{-3}$,m (b): 1-3) data from [24] for $2\langle a\rangle = 0.055 \cdot 10^{-3}$, $0.080 \cdot 10^{-3}$, and $0.128 \cdot 10^{-3}$ m; 4, 5) from [19] for $2\langle a\rangle = 0.051 \cdot 10^{-3}$ and $0.066 \cdot 10^{-3}$ m; 6) from [25] for $2\langle a\rangle = 0.13 \cdot 10^{-3}$ m, diameter of apparatus 0.152 m; 7, 8) results of present experiment for $2a = (0.1\text{–}0.16) \cdot 10^{-3}$ and $(0.16\text{–}0.2) \cdot 10^{-3}$ m; 9) generalizing correlation $Sh/\sqrt{PeRe} = 8.13 \cdot 10^{-2}\ Ar^{-1/4}$; 10, 16) results of present experiment for $2\langle a\rangle = 0.63 \cdot 10^{-3}$ and $0.25 \cdot 10^{-3}$ m; 12, 15) for $2a = (0.4\text{–}0.63) \cdot 10^{-3}$ and $(0.2\text{–}0.315) \cdot 10^{-3}$ m; 11, 13, 14) data from [25] for $2\langle a\rangle = 0.59 \cdot 10^{-3}$, $0.368 \cdot 10^{-3}$, and $0.29 \cdot 10^{-3}$ m, diameter of apparatus 0.152 m; 17) generalizing correlation $Sh/\sqrt{PeRe} = 6.7 \cdot 10^{-4}\ Ar^{1/2}$.

detailed experiments, in which the geometric characteristics of the bubble are monitored and the influence of the walls of the apparatus, etc., is established, is of undoubted theoretical interest. (It may be noted, in passing, that the influence of the walls was insignificant in the experiments described here: were this not the case, this influence would increase with rise in R, and it is clear from Fig. 1 that K is independent of R.)

Mass-Transfer Coefficients

The dependence of the mass-transfer coefficients on the bubble radius in beds of different particles and on the particle sizes of the bed for bubbles of different radii is shown in Fig. 2. The accuracy of the given mean values of k and k', corresponding to the 95% confidence level, was in all cases no less than 20%. It follows from the data of Fig. 2 that the dependence of k on the particle size is nonmonotonic (it has clearly expressed minima) and that k rises with increase in bubble radius. An episodic dependence of this type was also observed earlier (see [17-19], for example), but due attention was not paid to it. In addition, monotonic increase of k with rise in R contradicts all the known theoretical ideas regarding the character of the mass-transfer process (see the reviews in [6, 7, 17]), and plays a decisive role in its physical modeling.

It may be expected, a priori, that two fundamentally different mass-transfer mechanisms will exist. For large bubbles in beds of small particles (so that $u = u_*/\varepsilon < U$) there is a cloud of closed gas circulation, and mass transfer with the dense phase may only occur by means of diffusion through the outer boundary of the cloud, which is impenetrable to the gas clouds. For small bubbles in beds of large particles ($u > U$), this cloud does not exist, and convective transfer of impurities by the gas flows penetrating the bubble plays the main role. (In this discussion, no account is taken of the possible transfer of impurities by absorption on particles nor of the influence of the flow responsible for the change in volume of the bubble.) Although convection and diffusion (dispersion) may be equally important in intermediate cases ($u \sim U$), it is clearly expedient to begin by considering the two extreme situations individually.

Mass Transfer of Large Bubbles in Bed of Small Particles

The theory of diffusional mass transfer in minimally fluidized beds with no great influence of adsorption and nonsteady effects, in the case of large Peclet numbers, Pe, characteristic for bubbles in fluidized beds, was developed in [7]. It leads to the formula

$$k = C(UD/R)^{1/2}, \quad \mathrm{Sh}/\sqrt{\mathrm{Pe}} = C, \tag{7}$$

and a simple analytical expression may readily be derived for the coefficient C, depending on ε and f and also on the dimensionless ratios u/U and $\alpha u/D$. According to Eqs. (6) and (7), when $u/U \ll 1$, $k \sim R^{-1/4}$, which contradicts the data of Fig. 2a. This contradiction indicates that, in addition to the molecular diffusion in the intervals between the dense-phase particles and the convective dispersion due to mixing of the elementary gas jets in its pores, both of which are explicitly taken into account in [7], there is an additional, and evidently dominant, process of mixing of the gas volumes, leading to additional scattering of the impurity. This processes gives rise to yet another flow of diffusional type, with effective dispersion coefficient D_e, which is found to depend on the bubble dimensions. This means that the pulsations causing the appearance of this flow also depend on R, i.e., must be related to the oscillations of the bubble itself observed in the experiment. This viewpoint was adopted intuitively in [19]; these bubble pulsations are certainly connected with the "breakaway" of part of the cloud sometimes observed; the possible role of this in mass transfer was noted in [20].

To obtain the simplest order-of-magnitude relations, sufficient for present purposes, the well-known analogy between bubbles in a fluidized bed and gas bubbles in a single-phase ideal fluid is used. According to [21], the amplitude of pressure vibrations of an ideal fluid at the surface of a pulsating bubble is $\rho_d\omega^2 RS_n(1 + n)^{-1}$, where S_n is the amplitude of oscillations of the radius of a spherical bubble corresponding to the n-th spherical harmonic. It is clear that it must coincide in order of magnitude with the amplitude of the oscillations of the dynamic pressure of the medium surrounding the bubble, i.e., in the given case, with a quantity of the order of $\rho_d U^2(S_n/R)$. Hence, the following relationship for the frequency of the pulsations results*

$$\omega^2 \sim (1 + n)(U/R)^2 \sim R^{-1}. \tag{8}$$

The amplitude of oscillations of the bubble boundary must obviously be proportional to R, while the velocity of bubble motion in the normal direction must be proportional to the quantity $\omega R \sim \sqrt{R}$. In contrast to a bubble in drop liquids, the surface oscillations of a bubble in a fluidized bed must be accompanied by oscillations of the local gas flow penetrating this surface, and the pulsational velocity of the gas must be the same, in order of magnitude, as the pulsational velocity of the surface. Therefore, the following estimate is obtained for the coefficient D_e proportional to the product of the characteristic pulsational velocity and the effective mixing length† (see next page for footnote)

$$D_e \sim \omega R^2 \sim R^{3/2}. \tag{9}$$

In the general case, the total diffusional flux consists of some superposition of fluxes due to molecular diffusion, convective dispersion in the crossed pore space of the dense phase, and dispersion associated with the noted pulsations of the bubble itself. In most

*In the original theory in [21], the discussion concerns the oscillations of a bubble with its center of gravity fixed in the fluid, and the boundary condition for a normal stress at the surface of a bubble taking account of surface tension is used to determine the frequency of the pulsations. In the present case, the bubble is moving with respect to the disperse phase, and the effective surface tension coefficient at its surface is small [22], so that the pressure pulsations of the disperse phase are primarily associated with the oscillations of the bubble which it surrounds, and not with any surface effects.

practically realizable conditions, as in the experiments here described, the latter component evidently dominates, and impurity scattering due to the first two factors may in general be neglected in the first approximation.‡

The dependence of C from Eq. (7) on the bubble and bed characteristics corresponding to this process will now be considered. To simplify comparison with the data of other researchers, the defining dimensionless parameters chosen are the Reynolds and Archimedes numbers, which are traditionally introduced in this context. It is clear here that the intensity of mass transfer due to bubble pulsations and the resulting pulsations of the gas velocity must not depend on the kinematic viscosity of the gas. Therefore, C can only depend on the combination Re^2/Ar of the given numbers. On the other hand, a theory analogous to that developed in [7] leads to an expression $Q \sim \sqrt{D_e U}\, R^{3/2}$ for the total impurity flux from the bubble, and further to the relations $k \sim \sqrt{D_e U}\, R^{-1/2}$, $Sh \sim \sqrt{D_e U R}$ and $Sh/\sqrt{Pe} \sim \sqrt{D_e} \sim R^{3/4}$; Eqs. (6) and (9) are taken into account here. Since $Re \sim UR \sim R^{3/2}$, it follows that the quantity $Sh/\sqrt{Pe}$ must be proportional to $\sqrt{Re}$, and hence

$$Sh/\sqrt{PeRe} \sim Ar^{-1/4} \tag{10}$$

A dependence of the type in Eq. (10) is shown in Fig. 3a, together with experimental results for a bed of small particles and also data from [19, 24, 25].

It is readily evident that the use of the usual criteria Sh, Pe, Re, and Ar for the correlation of mass-transfer data in the present case is not natural. In fact, the dimensional parameters important for the processes are R, U (or g), the gas density ρ inside the bubble, and also the effective density ρ_d and the kinematic viscosity ν_d of the disperse phase. Hence, using the standard methods of dimensionality theory, it is found that

$$\frac{k}{U} = F(\rho/\rho_d,\ \nu_d/UR). \tag{11}$$

For granular beds fluidized by a gas, $\rho/\rho_d \approx 0$, and ν_d may be expressed as the product of some function of ε and the quantity $a u_*$ [26]. Taking into account that the porosity ε is approximately the same in the different experiments, the parameter $a u_*/UR$ may be used in place of ν_d/UR. The results of analysis of the data of Fig. 3a in the coordinates $k'/\sqrt{gR} \sim k'/U$ and $a u_*/UR$ are shown in Fig. 4. It is evident that the experimental points are in fact concentrated close to some universal curve. The parameter $a u_*/UR$ characterizes the ratio of the "viscous" stress in the disperse phase to the force of the dynamic pressure in it due to motion with velocity and length scales equal to U and R, respectively, while the dependence of the mass-transfer coefficient on this parameter reflects the influence of the effective viscosity of the disperse phase on the development in this phase of random oscillations of the bubbles, which are the primary cause of mass transfer in the given conditions.

Mass Transfer of Small Bubbles in a Bed of Large Particles

In this case, the washing out of impurities from the bubble is mainly due to the regular gas flow issuing from it [8]. To date, no rigorous theory of this washing out, taking diffusional effects into account, has been constructed, but in the limiting case of a bed of very large particles it may be asserted that $k \sim u_*$, and the dependence of k on the various

†The approximate description of the process of impurity scattering by the given pulsations as a random process of diffusional type is correct for the following reasons. First, it is obvious from Eqs. (6) and (8) that when $R \leqslant 0.05$ m the pulsation frequency $\omega \gtrsim \sqrt{g/R} \approx 15$–$20$ Hz, so that the bubble is able to perform a large number of oscillations in the time interval of interest, of the order of seconds or more (no less than 8-10 oscillations in the time for the bubble to cross the distance between the sensors, amounting to no less than 0.5 sec in the experiments here described). Second, the bubble simultaneously performs oscillations corresponding to a large number of different spherical harmonics, while the phases of these oscillations are random and mutually independent. Third, what is of practical interest is the mass-transfer coefficient associated with one bubble in a large group of other bubbles, and not the coefficient for any single specific bubble (the experiments described also satisfy this condition: they are concerned with the coefficient k obtained by averaging the results of many experiments with different bubbles).

‡But situations are also known where the bubble pulsations are retarded by unknown factors and molecular diffusion and convective dispersion play the main role. This situation was realized in the experiments of [23], which are described fairly well by the theories of [7].

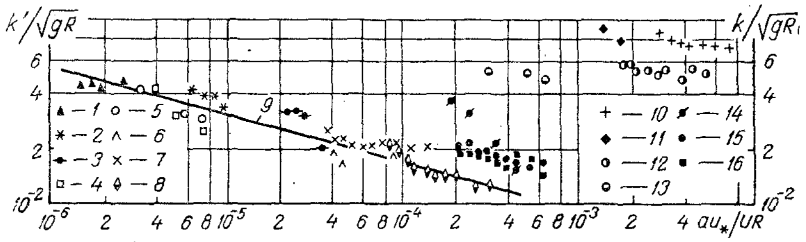

Fig. 4. Dependence of the dimensionless rates of bubble mass transfer $k'/\sqrt{gR}$, $k/\sqrt{gR}$ on the parameter complex (au_*/UR) (the notation for the experimental points is the same as in Fig. 3): 1-8) values of $k'/\sqrt{gR}$ for particles with $2\langle a\rangle < 0.2\cdot 10^{-3}$ m; 10-16) experimental values of $k/\sqrt{gR}$ for particles with $0.2\cdot 10^{-3} < 2\langle a\rangle < 0.63\cdot 10^{-3}$ m; 9) generalizing correlation $k'/\sqrt{gR} = 1.7\cdot 10^{-3}\cdot (au_*/UR)^{-1/4}$.

parameters is determined mainly by the dependence of the quantity u_* on them. Thus, in the range of bed-particle diameters from $2.5\cdot 10^{-4}$ to $6.3\cdot 10^{-4}$ m, according to the data of Table 1, the velocity u_* is approximately proportional to $a^{3/2}$, i.e., the following dependence would be expected

$$\mathrm{Sh}/\sqrt{\mathrm{PeRe}} \sim \sqrt{\mathrm{Ar}}. \tag{12}$$

This dependence, and also the experimental data for beds of large particles, are shown in Fig. 3b. As in the case of a diffusional transfer mechanism, it makes more sense to analyze the experimental results in terms of the coordinates $k/\sqrt{gR}$, au_*/UR. The corresponding experimental points are also shown in Fig. 4; it seems that they also lie on a single curve, although the data obtained are inadequate for a final judgement as to the universality of this curve.

The results of the present work may be used to calculate interphase mass transfer in equipment with fluidized beds.

NOTATION

a, $\langle a\rangle$, radius and mean radius of particles; C—$\mathrm{Sh}/\sqrt{\mathrm{Pe}}$; c, impurity concentration in bubble; c_1, c_2, values of c at the levels of the sensors; D, D_e, coefficients of molecular diffusion and effective dispersion; F, function in Eq. (11); f, fraction of the volume of a sphere of radius R occupied by the bubble tail; g, acceleration due to gravity; K, velocity coefficient; k, k', mass-transfer coefficients referred to unit surface area of bubble and cloud; l, vertical coordinate; Δl, δl, distance between sensors and between pulses on recording tape; Q, impurity flux from bubble; R, R', and R*, radii of frontal part of bubble and cloud and half the vertical bubble dimension; S, bubble surface area; S_n, amplitude of pulsations of R corresponding to the n-th spherical harmonic; t, time; U, bubble rise velocity; $u = u_*/\varepsilon$; u_*, minimum-fluidization velocity; V, bubble volume; v, speed of recording tape; δ, mean pulse width; ε, porosity of dense phase; ν, ν_d, kinematic viscosity of gas and dense phase; ρ_p, ρ, ρ_d, density of particles, gas, and disperse phase; ω, pulsation frequency; $\mathrm{Sh}=\dfrac{kR}{D}$; $\mathrm{Pe}=\dfrac{UR}{D}$; $\mathrm{Re}=\dfrac{UR}{\nu}$; $\mathrm{Ar}=\dfrac{(2a)^3 g}{\nu^2}\,\dfrac{\rho_d}{\varepsilon\rho}$

LITERATURE CITED

1. P. N. Rowe, "Experimental investigation of gas-bubble properties," in: Fluidization [in Russian], Khimiya, Moscow (1974), pp. 122-169.
2. V. A. Borodulya, V. I. Dikalenko, and V. I. Kovenskii, "Heat and mass transfer between bubbles and dense phase in fluidized bed," in: Heat and Mass Transfer in Multiphase Multicomponent Systems [in Russian], ITMO Akad. Nauk BSSR, Minsk (1978), pp. 3-17.
3. R. Toéi, R. Matsuno, Kh. Khotta, and M. Oiti, "Influence of adsorptional capacity or specific heat of particles on gas transfer or heat transfer of bubbles with emulsion phase in a bed of solid particles fluidized by a gas," in: Heat and Mass Transfer [in Russian], Vol. 5, Naukova Dumka, Kiev (1972), Part 1, pp. 206-229.
4. P. N. Rowe and R. Matsuno, "Single bubbles injected into a gas fluidized bed and observed by x rays," Chem. Eng. Sci., 26, No. 6, 923-935 (1971).

5. Yu. A. Buevich, "A model of bubbles rising in a fluidized bed," Int. J. Multiphase Flow, 2, No. 2, 337-351 (1975).
6. Yu. A. Buevich and A. N. Deryabin, "Mass transfer from single bubble to dense phase of fluidized bed at large Peclet numbers," Inzh.-Fiz. Zh., 38, No. 2, 201-210 (1980).
7. V. A. Borodulya, Yu. A. Buevich, and V. I. Dikalenko, "Mass transfer of a single bubble in a minimally fluidized granular bed," Inzh.-Fiz. Zh., 39, No. 2, 323-333 (1980).
8. J. F. Davidson and D. Harrison (eds.), Fluidization, Academic Press (1971).
9. I. F. Davidson, R. C. Paul, I. S. Smith, and H. A. Duxbury, "The rise of bubbles in a fluidized bed," Trans. Inst. Chem. Eng., 37, No. 6, T323-T328 (1959).
10. D. Harrison and L. S. Leung, "The rate of rise of bubbles in fluidized beds," Trans. Inst. Chem. Eng., 40, No. 3, T146-T151 (1962).
11. P. N. Rowe and B. A. Partridge, "An x-ray study of bubbles in fluidized beds," Trans. Inst. Chem. Eng., 43, No. 5, T157-T175 (1965).
12. R. Toei, R. Matsuno, H. Kojima, Y. Nagai, K. Nakagawa, and S. Yu, "Behavior of bubbles in the gas—solid fluidized beds," Mem. Fac. Eng., Kyoto Univ., 27, Pt. 4, 475-489 (1965).
13. H. Kobayashi, F. Arai, and T. Chiba, "Behavior of bubbles in a gas—solid fluidized bed," Kagaku-kogaku (abr. ed.), 4, No. 1, 147-150 (1966).
14. W. H. Park, W. K. Kang, C. E. Capes, and G. L. Osberg, "The properties of bubbles in fluidized beds of conducting particles as measured by an electroresistivity probe," Chem. Eng. Sci., 24, No. 5, 851-865 (1969).
15. G. Donsi, L. Massimilla, S. Cresctelli, and G. Volpicelli, "Solid flow pattern at the wall of a fluidization column induced by single bubbles," Powder Technol., 6, No. 4, pp. 217-224 (1972).
16. R. R. Cranfield and D. Geldart, "Large particle fluidization," Chem. Eng. Sci., 29, No. 4, 935-947 (1974).
17. A. A. H. Drienkenburg and K. Rietema, "Gas transfer from bubbles in a fluidized bed to the dense phase. I. Theory," Chem. Eng. Sci., 27, No. 10, 1765-1774 (1972).
18. A. A. H. Drienkenburg and K. Rietema, "Gas transfer from bubbles in a fluidized bed to the dense phase. II. Experiment," Chem. Eng. Sci., 28, No. 1, 259-273 (1973).
19. J. Hoebink and K. Rietema, "Mass transfer between single bubbles and the dense phase in gas—solid fluidization," in: Particle Technology, Nuremburg (1977), pp. K51-K65.
20. R. Toei, R. Matsuno, H. Miyagawa, K. Nishitani, and Y. Komagawa, "Gas transfer between a bubble and the continuous phase in a gas—solid fluidized bed," Int. Chem. Eng., 9, No. 2, 358-364 (1969).
21. H. Lamb, Hydrodynamics, Dover (1932).
22. Yu. A. Buevich and Yu. P. Gupalo, "Rupture surfaces in disperse systems," Prikl. Mat. Mekh., 34, No. 4, 722-734 (1970).
23. R. Chiba and H. Kobayashi, "Gas exchange between the bubble and emulsion phases in gas—solid fluidized beds," Chem. Eng. Sci., 25, No. 9, 1375-1385 (1970).
24. L. Davies and J. F. Richardson, "Gas interchange between bubbles and the continuous phase in a fluidized bed," Trans. Inst. Chem. Eng., 44, No. 8, T293-T305 (1966).
25. G. K. Stephens, R. J. Sinclair, and O. E. Potter, "Gas exchange between bubbles and dense phase in a fluidized bed," Powder Technol., 1, No. 3, 157-166 (1967).
26. Yu. A. Buevich, V. N. Varygin, and E. N. Prozorov, "Small-scale mixing in a bed of small particles fluidized by a gas," Inzh.-Fiz. Zh., 38, No. 5, 836-846 (1980).